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144. Proposed F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

In a circular park 400 feet in diameter are four *equal* circular ponds of *variable* diameter. What is the probability that a sightless person walking in a straight line from the center of the park will step into a pond?

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

In this solution we shall assume that a random radius (from center C of circular park of radius r) does not intersect more than one pond on the same side of the center. The required chance, p, then is four times the chance that a random radius will intersect a pond lying wholly upon the surface of a quadrant of the given circle.

Let P be center of pond, x its radius, θ=angle between tangents from C to pond, φ=angle at P between radii to points of contact. Let $CP=y$. Then $\theta=2\sin^{-1}(x/y)$, $\varphi=\frac{1}{2}\pi-2\sin^{-1}(x/y)$. The limits of x are 0 and $r(\sqrt{2}-1)=x'$; of y, $x\sqrt{2}$ and $r-x$.

$$\therefore p=4\frac{\int_0^{x'}\int_{x\sqrt{2}}^{r-x}\theta\varphi\,dxydy}{\int_0^{x'}\int_{x\sqrt{2}}^{r-x}2\pi\varphi\,dxydy}.\quad 2\pi\int_0^{x'}\int_{x\sqrt{2}}^{r-x}\varphi dxydx$$

$$=\frac{\pi}{2}\int^{x'}\left[\pi(r-x)^2-4(r-x)^2\sin^{-1}\left(\frac{x}{r-x}\right)-4x\sqrt{(r^2-2rx)}+4x^2\right]dx$$

$$=\frac{\pi r^3}{18}(3\pi-28+16\sqrt{2}).$$

$$\therefore p=\frac{72}{\pi r^3(3\pi-28+16\sqrt{2})}\int_0^{x'}\int_{x\sqrt{2}}^{r-x}\theta\varphi\,dxydy$$

$$=\frac{36}{\pi r^3(3\pi-28+16\sqrt{2})}\int_0^{x'}\left[\pi(r-x)^2\sin^{-1}\left(\frac{x}{r-x}\right)+\pi x^2+\pi x\sqrt{(r^2-2rx)}\right.$$

$$\left.-4(r-x)^2\left\{\sin^{-1}\left(\frac{x}{r-x}\right)\right\}^2-9x\sqrt{(r^2-2rx)}\sin^{-1}\left(\frac{x}{r-x}\right)-8x^2\log\left(\frac{r-x}{x\sqrt{2}}\right)\right]dx$$

$$=\frac{4}{(3\pi-28+16\sqrt{2})}\left[4\sqrt{2}-3+\frac{8}{\pi}-\frac{8\sqrt{2}}{\pi}+\frac{8}{\pi}\log(2-\sqrt{2})\right].$$

MISCELLANEOUS.

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138. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Find an invariant of the *third degree* in the coefficients of a ternary cubic.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

Let $U=ax^4+by^4+cz^4+4(a_2x^3y+a_3x^3z+b'_3y^3z+b_1y^3x+c_1z^3x+z_3z^3y)$

$$+6(dy^2z^2+ez^2x^2+fx^2y^2)+12xyz(lx+my+nz)$$

be the ternary quartic. The required invariant written out in full is

$$\frac{d^4u}{dx^4}\cdot\frac{d^4u}{dy^4}\cdot\frac{d^4u}{dz^4}-4\left(\frac{d^4u}{dx^4}\cdot\frac{d^4u}{dy^3\,dz}\cdot\frac{d^4u}{dz^3\,dy}+\frac{d^4u}{dy^4}\cdot\frac{d^4u}{dz^3\,dx}\cdot\frac{d^4u}{dx^3\,dz}+\frac{d^4u}{dz^4}\cdot\frac{d^4u}{dx^3\,dy}\cdot\frac{d^4u}{dy^3dx}\right)$$

$$+3\left[\frac{d^4u}{dx^4}\left(\frac{d^4u}{dy^2\,dz^2}\right)^2+\frac{d^4u}{dy^4}\left(\frac{d^4u}{dx^2\,dz^2}\right)^2+\frac{d^4u}{dz^4}\left(\frac{du^4}{dx^2\,dy^2}\right)\right]$$

$$+4\left(\frac{d^4u}{dx^3dy}\cdot\frac{d^4u}{dy^3\,dz}\cdot\frac{d^4u}{dz^3\,dx}+\frac{d^4u}{dx^3\,dz}\cdot\frac{d^4u}{dy^3dx}\cdot\frac{d^4u}{dz^3\,dy}\right)-12\left(\frac{d^4u}{dx^3dy}\cdot\frac{d^4u}{dxdydz^2}\cdot\frac{d^4u}{dy^2\,dz^2}\right.$$

$$+\frac{d^4u}{dx^3\,dz}\cdot\frac{d^4u}{dx\,dy^2dz}\cdot\frac{d^4u}{dy^2\,dz^2}+\frac{d^4u}{dy^3dx}\cdot\frac{d^4u}{dx\,dy\,dz^2}\cdot\frac{d^4u}{dz^2\,dx^2}+\frac{d^4u}{dy^3\,dz}\cdot\frac{d^4u}{dx^2\,dy\,dz}\cdot\frac{d^4u}{dz^2\,dx^2}$$

$$\left.+\frac{d^4u}{dz^3\,dx}\cdot\frac{d^4u}{dx\,dy^2\,dz}\cdot\frac{d^4u}{dx^2\,dy^2}+\frac{d^4u}{dz^3\,dy}\cdot\frac{d^4u}{dx^2\,dy\,dz}\cdot\frac{d^4u}{dx^2\,dy^2}\right)+12\left(\frac{d^4u}{dx^2dydz}\cdot\frac{d^4u}{dy^3dx}\cdot\frac{d^4u}{dz^3\,dx}\right.$$

$$\left.+\frac{d^4u}{dx\,dy^2dz}\cdot\frac{d^4u}{dz^3\,dy}\cdot\frac{d^4u}{dx^3dy}+\frac{d^4u}{dx\,dy\,dz^2}\cdot\frac{d^4u}{dx^3\,dz}\cdot\frac{d^4u}{dy^3\,dz}\right)+12\left[\frac{d^4u}{dy^2\,dz^2}\cdot\left(\frac{d^4u}{dx^2\,dy\,dz}\right)^2\right.$$

$$\left.+\frac{d^4u}{dz^2\,dx^2}\cdot\left(\frac{d^4u}{dx\,dy^2\,dz}\right)^2+\frac{d^4u}{dx^2\,dy^2}\cdot\left(\frac{d^4u}{dx\,dy\,dz^2}\right)^2\right]+6\frac{d^4u}{dy^2\,dz^2}\cdot\frac{d^4u}{dy^2\,dx^2}\cdot\frac{d^4u}{dx^2\,dz^2}$$

$$-12\frac{d^4u}{dx^2\,dy\,dz}\cdot\frac{d^4u}{dx\,dy^2\,dz}\cdot\frac{d^4u}{dx\,dy\,dz^2}.$$

$\therefore abc-4(ab_3c_2+bc_1a_3+ca_2b_1)+3(ad^2+be^2+cf^2)+4(a_2b_3c_1+a_3b_1c_2)$
$-12(a_2nd+a_3md+b_1ne+b_3le+c_1mf+c_2lf)+12(lb_1c_1+mc_2a_2+na_3b_3)+12(dl^2$
$+em^2+fn^2)+6def-12lmn.$

139. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Given the roots of a binary cubic, to find the roots of its two inbependent covariants.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, West Va.

Let $ax^3+3bx^2y+3cxy^2+dy^3=0$ be the cubic. The two independent covariants are,